## Calculus in Higher Dimensions

Background

This topic deals with extending concepts such as limits, continuity, differentiation, and integration, studied in first year calculus, to functions of several variables. Topics covered:

- Continuity of functions of several variables.

- Limits, partial derivatives, gradients, directional derivatives, divergence, and curl and apply these concepts to problem solving.

- Nature of extrema and optimization problems using Lagrange multipliers.

- Determine double and triple integrals and use them to calculate areas and volumes.

- Determine line, surface and flux integrals and apply the theorems of Green, Stokes and Gauss, which relate these types of integrals.

The Study Guide Splits the above topics into three distinct units:  
Basic Concepts

* Preliminaries (Sets, Relations, Implications, Symbols)
* N-dimensional Euclidian space (R, dot products, Norm, Distance, Unit Vectors, Basis Vectors, Angle between vectors, Cross Product, Lines, Subsets)
* Functions (visualisation, Rn-Rp)

Differentiation

* Limits and Continuity (R-R functions, Rn-R functions, Real Valued functions, Limits along curves, Vector Valued functions, Continuity)
* Derivatives Real Valued functions (One Variable)
* Derivatives Vector Valued functions (Chain Rule, Piecewise smooth curves)
* Derivatives Real Valued functions (Several Variables) (Rn-R functions, Gradient of Rn-R functions, Differentiability of Rn-R functions, Chain Rule, Directional Derivatives Rn-R functions, Potential Functions, Higher order Partial Derivatives)
* Derivatives of Vector Field
* Taylor Polynomials (R-R functions, Rn-R functions)

Integration

* Single Integrals
* Double Integrals
* Triple Integrals
* Line Integrals
* Surface Integrals
* Flux Integrals
* Theorems (Green, Gauss, Stokes)

**Lesson 1**

dimensional Euclidean Space

: One-dimensional space

Can be represented as a straight line

Corresponds with the set of real numbers

Written as the ordered tuple

: Two-dimensional space

Can be represented geometrically as a plane

Corresponds with two mutually perpindicular copies of , called the axis and axis

Origin denoted , is the point

Written as the ordered pair

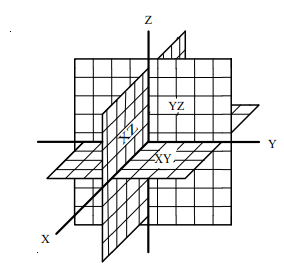
: Three-dimensional space

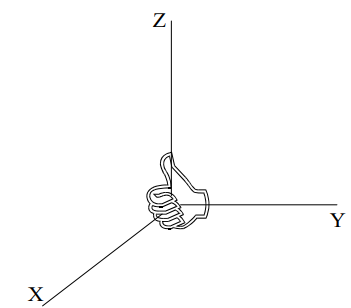
Can be represented geometrically as a plane

Corresponds with three mutually perpindicular copies of , called the axis, axis and axis

Origin denoted , is the point

Written as the ordered tuple

 *right hand rule coordinate planes in*



**Lesson 2**

Vectors in

The standard geometric definition of vector is as something which has direction and magnitude but not position. Since vectors have no position, we may place them wherever is convenient.

A vector in is a tuple

*Essentially a matrix*

written as: and

**Vector Addition**

**Vector Subtraction**

**Scalar Multiplication**

Two vectors and are parallel if they are scalar

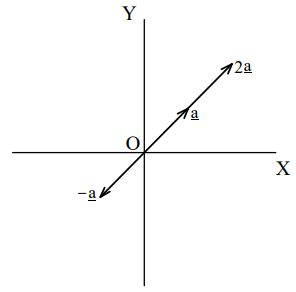
multiples of one another

where

*Algebraic definition of parallel vectors in*

Example: Which of the following vectors parallel?

*This number should not have a remainder*



**Cross Product**

Two vectors and are perpendicular

their scalar product is equal to zero

Example: a vector perpendicular to the

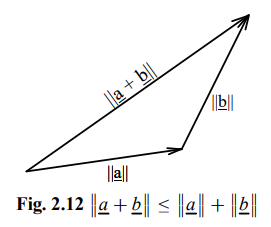
vectors and

**Dot Product**

The Norm in (length in )

*Dot product is a real number (scalar product)*

Example: the length of the vector



**Leson 3**

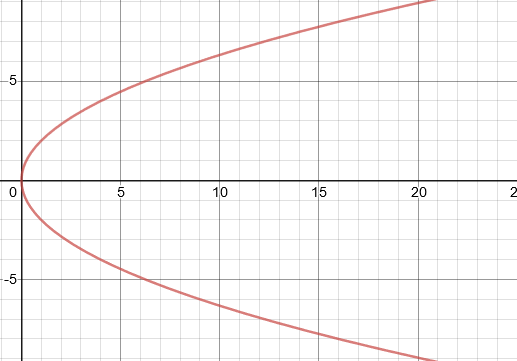
Parametric Equations

A parametric equation is where the x and y coordinates are both written in terms of another letter. This is called a parameter and is usually given the letter t or . ( is normally used when the parameter is an angle, and is measured from the positive x-axis.)

Example: Plot the graph of

*Chose some random values for t*

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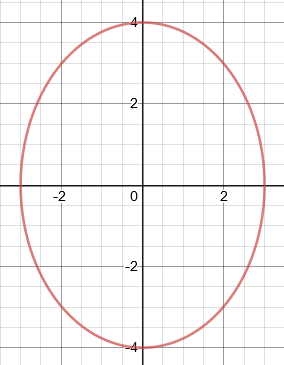
Desmos Graphing Calculator

*Don’t simplify further,*

Example: Plot the graph of

*Chose values for t which will give a good range of points for*

|  |  |  |  |  |  |  |  |  |  |
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*Try write the equation in the form ,*

Desmos Graphing Calculator

ASS1 Q1a:

Find the point of intersection , If there is one, of the following lines:

If and intersect, there is a point that lies on both lines. There must be such that:

*and are only distinguished for legibility*

If and intersect, there is a point that lies on both lines. There must be such that:

If and intersect, there is a point that lies on both lines. There must be such that:

Therefore, , which is a point on and

If and are in the plane they describe,